Chaos

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Cite as: Chaos **28**, 093108 (2018); https://doi.org/10.1063/1.5040714
Submitted: 19 May 2018 . Accepted: 30 August 2018 . Published Online: 13 September 2018

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Effects of external forcing on evolutionary games in complex networks

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(Received 19 May 2018; accepted 30 August 2018; published online 13 September 2018)

How did cooperation evolve in a complex network is an intensely investigated problem. Many mechanisms that promote cooperation have been proposed within the framework of the evolutionary game theory. Motivated by the fact that people in society or even a certain group are often controlled by a variety of simple rules, we present an external forcing mechanism to analyze the underlying reasons of widespread cooperation in this paper. In detail, we model the agents on a simple regular network, on which the learning method is controlled by external forcing mechanism, and prisoner's dilemma has been applied to describe the interaction of agents. By conducting large-scale Monte Carlo simulations, we can easily draw a conclusion that this mechanism can promote cooperation efficiently. In addition, we also show that the proposed mechanism is effective for the cooperation promotion for other game models, such as snowdrift game and multigames. Taken together, the mechanism of external forcing on the evolutionary game is a strong promoter of cooperation even under a severe temptation condition, which has a practical significance and will provide new insight into the analysis and control of cooperative strategy in the complex network for the further research. *Published by AIP Publishing.* https://doi.org/10.1063/1.5040714

The emergence of cooperation among agents in sociality has been an intensely investigated problem for years. Typically, it was often studied within the framework of evolutionary game theory. Many effective mechanisms have been proved to be useful for cooperation when they can promote cooperators to form into clusters. Here, the cooperators' clusters can resist the invasion of defectors. In this paper, we present an external forcing mechanism to analyze the underlying reasons of widespread cooperation. In detail, we model the agents on a simple regular network, on which the learning method is controlled by an external forcing mechanism. Numerical results demonstrate that the proposed mechanism can promote cooperation efficiently. Moreover, our results also demonstrate that the proposed mechanism is robust for different kinds of game models, such as snowdrift game and multigames. Therefore, we can conclude that the mechanism of external forcing on the evolutionary game is a strong promoter of cooperation even under a severe temptation condition, which has a practical significance and will provide new insight in resolving the cooperation puzzle for further research.

I. INTRODUCTION

In reality, there are a large number of agents, who often interact with each other for the purpose of a limited resource, and hence the conflict between different agents occurs. According to Darwin's theory, most of the agents will select the defection strategy at the end. However, cooperation phenomenon is ubiquitously observed in the real world and in different organizations, such as microorganisms, animal groups, and human societies. Therefore, the problem called social dilemma occurs.[1,2] Mathematically, the topology of interaction between these agents is often modelled by the complex network,[3–9] such as regular lattice, small-world networks, scale-free networks, and so on. The interaction, on the other hand, is often described under the framework of the evolutionary game theory.[10–13] Thus, the subject of networked evolutionary game appears, and has attracted much attention from multiple disciplines such as biology, physics, mathematics, and engineering.[13–18]

For the networked evolutionary game theory, it assumes that bounded rational agents are interacting repeatedly, and all the agents will update their strategies according to some adaptation and learning rules.[10] Clearly, the dynamics of the networked evolutionary game includes three fundamental elements: (1) the network model, (2) the strategic decision-making model, and (3) the game model. The pioneering work of the network model for the social dilemma was put forward by Nowak and May in 1992, which shows that the regular lattice can induce the emergence of cooperation in a prisoner's dilemma environment by network reciprocity.[19,20] Along with this line, the network model of agents have been extensively studied, including the scale-free networks,[21] hierarchy networks,[22,23] and interdependent networks,[24–27] and these network models have been proved to be valuable for solving the dilemma to some extent.

For game model, the repeated prisoner's dilemma and the snowdrift game served as two paradigms for expressing a social dilemma in the evolutionary game theory.[28–34] In addition, as agents in the complex network may have heterogeneous perceptions for the social dilemma, the multigame model also has been proposed.[35,36] Besides, the

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mechanisms related to strategic decision-making for resolving the social dilemma have been introduced, such as reputation,[37–39] punishment and reward,[40,41] and mobility of agents,[42,43] to name but a few.

Generally, the previous works mainly consider that agents in the complex network can choose their strategies freely. However, the situation is not consistent with reality. In fact, there are always a variety of simple rules to restrict the agents' activity in the society or even a certain group. Therefore, we introduce a mechanism that the agent can update his strategy by a simple external force. In detail, we model the population on a simple regular network, on which the learning method is controlled by the external forcing mechanism, and prisoner's dilemma, snowdrift game, as well as multigames are used to describe the interaction of agents. To the best of our knowledge, few papers discuss the effect of the external forcing mechanism on the evolutionary dynamics, therefore, it is meaningful for solving the puzzle of social dilemma.

The rest of the paper is organized as follows: Sec. II introduces the mathematical model of the evolutionary game dynamics in the complex network. Section III explores the emergence of cooperation in the complex network by the proposed external forcing mechanism according to the large-scale Monte Carlo simulations. The concluding remarks are given in Sec. IV.

II. THE MODEL

As we all know, our society may operate smoothly and harmoniously because of legal documents that everyone abides by. These documents with only a few lines of words have played an important role in promoting the society towards the development of imagination as a kind of external force. Motivated by this fact, we present an external forcing mechanism to analyze the underlying reason for widespread cooperation in the complex network in this paper. Mathematically, we model the agents in a simple regular network, on which the learning method is controlled by the external forcing mechanism, and evolutionary prisoner's dilemma, snowdrift game and multigames have been applied to describe the interaction of agents.

In the evolutionary game model, each agent in the game could choose cooperation or defection. Here, the mutual cooperation obtains the reward R, the mutual defection yields the punishment P, and the mixed choice gives cooperator the sucker's payoff S and defector the temptation T. The payoff ranking of traditional prisoner's dilemma game is $T > R > P > S$, so we know that punishment P should be larger than sucker's payoff S. It is necessary to emphasize that we adopt $T = b, R = 1, S = 0$, and $P = 0$ (namely, weak prisoner's dilemma), but the results are robust and could be observed in the full parameterized space. For the snowdrift game, cost-to-benefit ratio r $(0 < r < 1)$ is imported, where the expected benefits of trying to cooperate or betray are the same as an opponent. Then, the reward for mutual cooperation $R = 1$, the temptation to defect $T = 1 + r$, the sucker's payoff $S = 1 - r$, and the punishment for mutual defection $P = 0$. So, the payoff matrices of the above games are shown, respectively, as

$$P_{PDG} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{1}$$

$$P_{SDG} = \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix}. \tag{2}$$

For multigames, the payoff matrix is shown as Eq. (3). Here, the same value of S represents individuals with the same social dilemma. Especially, all the agents choose to apply $S = +\Theta$ or $S = -\Theta$ with equal probability. Similarly, one-half of the agents play the traditional prisoner's dilemma, while the other half partly plays the snowdrift game. The distributions of positive and negative S are the same, which indicate that the average overall payoff matrices return the weak prisoner's dilemma. Therefore, the multigames are convenient for comparisons with the case of pure weak prisoner's dilemma. Primarily, we deem that the agents, who participate in multigames, do not change their payoff matrix once choosing their game model at the very beginning.

$$P_{MG} = \begin{pmatrix} 1 & S \\ b & 0 \end{pmatrix}. \tag{3}$$

We study evolutionary cooperation game with agents occupying each vertex on an $N = L \times L$ square lattice with periodic boundary. Each agent opts to act as a cooperator or a defector with equal probability. And we utilize Moore neighborhood, also known as, every agent interacts with its $k = 8$ nearest neighbors. Agents would get their corresponding payoffs after each round of evolutionary game. We deem that the payoffs are simultaneously accumulated by interacting with their closest neighbors.

Agents interact with their neighbors in each game round, and each one will update its strategies, after they all have grabbed payoffs in this round. At this stage, we could introduce the external forcing mechanism that enables the population to have considerable cooperation even in high temptation T situations. Just as there are still behaviors that do not abide by the rules under the conditions of regular constrains, the constrains of external forces can only be effective to some extent. In detail, we incorporated the external forcing mechanism by the following setting: each agent has the probability Pr $(0 \le Pr \le 1)$ to learn the strategy C (to be a cooperator in the next round), and then has the probability $1 - Pr$ to choose the strategy of one of his neighbors. Assume that one agent i is not controlled by the external forcing mechanism, he will choose one of his neighbors j randomly to update his strategy. Specifically, when $Pr = 0$, which is reduced to the traditional case, namely, no external force for agents in the complex network. According to Fermi rule, the probability of agent i to opt agent j's strategy depends on their payoff difference:[44]

$$W\left[s_i(t+1) = s_j(t)\right] = \frac{1}{1 + \exp\left\{\left[\Pi_i(t) - \Pi_j(t)\right]/K\right\}}, \tag{4}$$

where K indicates the uncertainty of strategy learning process,[45] and the uncertainty is often related to the strategy adoption process, serving to avoid trapped conditions and enabling smooth transitions towards stationary states. $\Pi_i(t)$ is

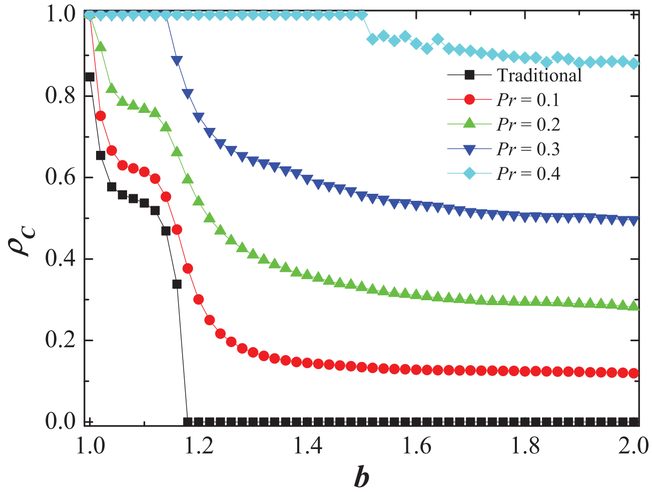

FIG. 1. Proportion of cooperators ρ_C on the square lattice changes with temptation b in the prisoner's dilemma, as acquired with different strengths of external forcing mechanism represented by Pr. Presented results are obtained for $K = 0.1$. The proportion of cooperators ρ_C at the stable state decreases as b increases, but faster when external forcing mechanism is missing.

the payoff of agent i in the tth step, which can be calculated as follows:

$$\Pi_i(t) = \sum\nolimits_{j \in \Omega_i} s_i^T(t) P_M s_j(t)\,. \tag{5}$$

Here, P_M is the payoff matrix of game model. s_x is the strategy of agent x, when the agent x chooses cooperation, $s_x = [1, 0]^T$, on the contrary, when the agent x chooses defection, $s_x = [0, 1]^T$.

We simulate the evolutionary game under the classic Monte Carlo simulation scheme, and the simulation results are obtained from the regular square lattice with $N = 100 \times 100$. Near phase transition points we have further increased the system size to avoid accidental extinctions and to ensure suitable accuracy. The proportion of cooperators through the whole

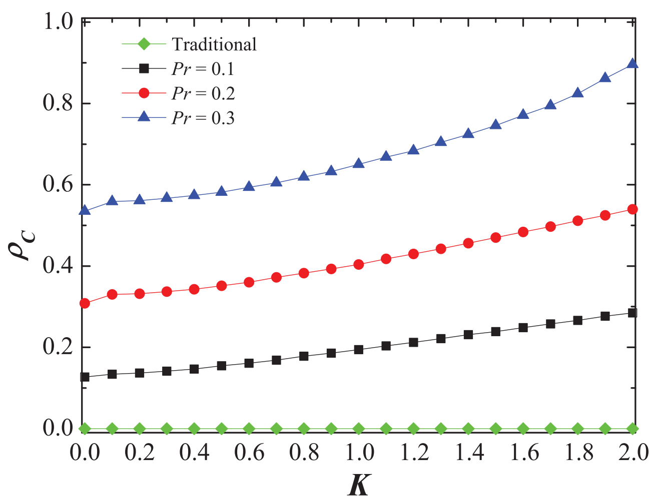

FIG. 2. Proportion of cooperators ρ_c on the square lattice changes with uncertainty of the strategy selection K in the prisoner's dilemma game, as acquired with different strengths of external forcing and different temptation $b = 1.5$. The higher the K, as well as the more chaotic the environment, the higher the proportion of cooperators. At the same K value, the stronger external forcing, the more effective to maintain the proportion of cooperators for the slope more steep.

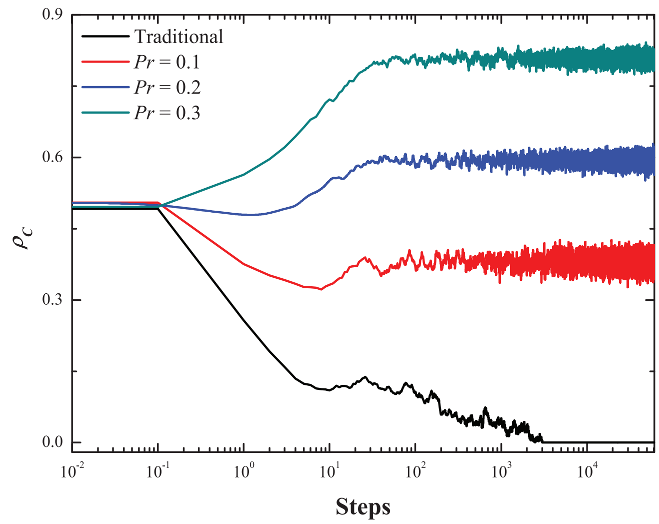

FIG. 3. Time courses of the proportions of cooperators ρ_C in the prisoner's dilemma game on the square lattice, as acquired without and with different intensive external forcing. Presented results are obtained for $K = 0.1$, $b = 1.18$. Defectors occupy the major position for the curve decreases in the early period stage; however, the external forcing re-raised the proportion of cooperators and kept it till the stable state, in contrast, defectors invade this testing group without control from the external forcing. And the cooperators occupy the major position when the external forcing reach a certain level as the curve keeps climbing at the early stage when $Pr = 0.3$.

network is represented by ρ_c, which is used as an evaluation standard for the cooperation of the population. We get the proportion ρ_c by calculating the average value from the last 1000 steps after enough long transient is discarded, namely, when the system steps into the stationary state. In order to guarantee higher accuracy, the final results are obtained from the average of 20 independent experiments.

III. NUMERICAL RESULTS

The impact of external forcing mechanism on maintaining the proportion of cooperation was detected through the Monte Carlo simulation method in the scenarios where the above game model occurred. There are many previous works[36,46] indicating that cooperative agents can coexist with defection agents in spatial populations due to network reciprocity. At the same time, the introduction of external forcing mechanism could make a significant impact on the changes in cooperative strategy. Now, we exploit the effect of external forcing mechanism on cooperative strategy evolution.

As we all know, cooperators are extinct when the temptation to defect $T = b$ increases by a very small amount (i.e., 1.04) in a spatial prisoner's dilemma. Thus, it is very important to propose a new mechanism to maintain the proportion of cooperation in the same circumstances. As it is shown in Fig. 1, the proportion of cooperators ρ_c at the stable state depends on the temptation to defect $T = b$ in prisoner's dilemma. When agents play the evolutionary game with their neighbors and lean the strategy on their own, the model degenerated to classical prisoner's dilemma game and the evolution of cooperative strategy totally depend on the spatial reciprocity. In this case, the level of cooperation is remarkably low and declines quickly. Five polylines represent the variation of the proportion of cooperation ρ_c with increasing

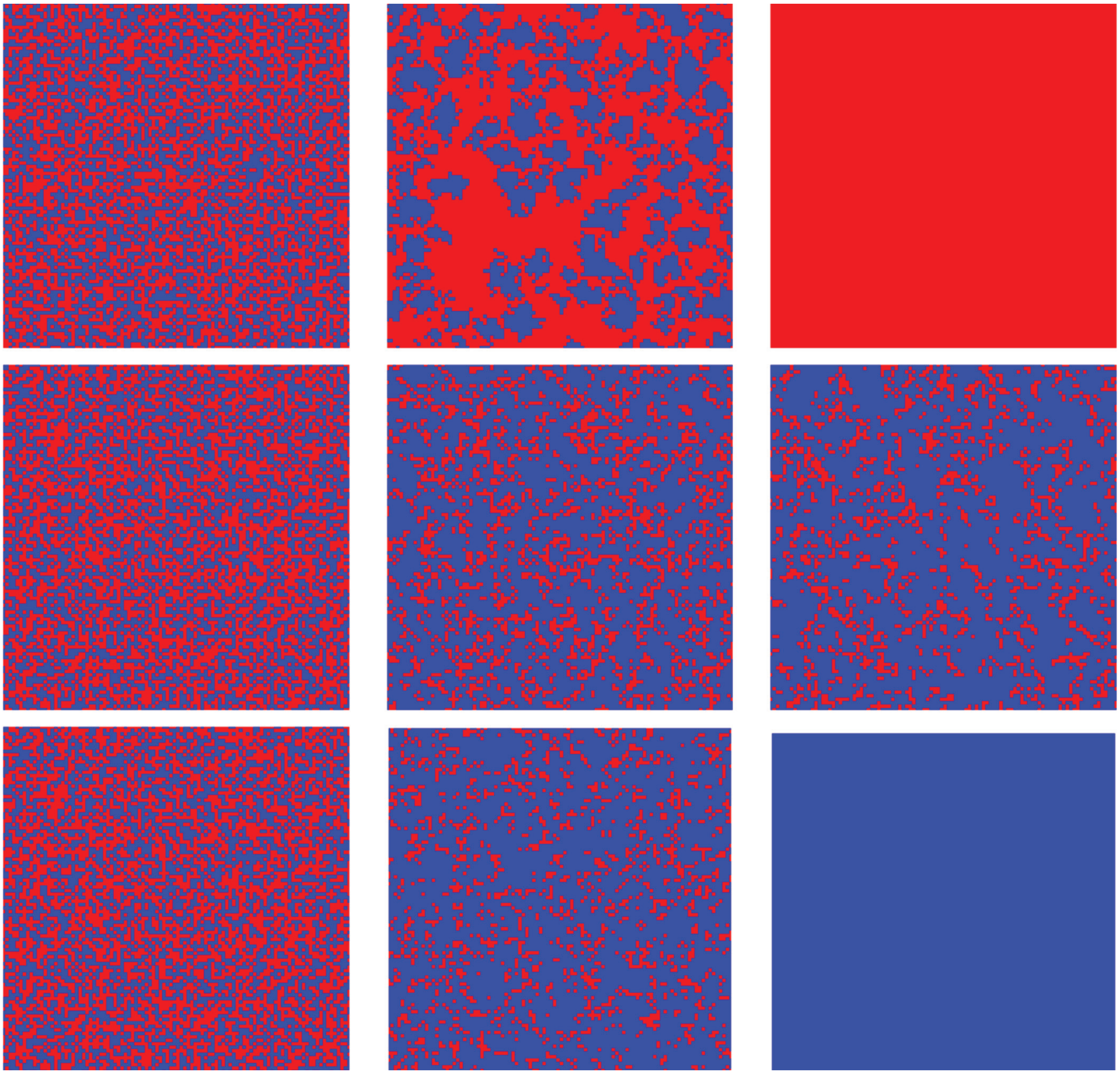

FIG. 4. Typical snapshots of the distribution of strategy in steps 0, 8, and 11 000. All results are obtained for $K = 0.1$, $b = 1.2$. From top to bottom, the strength of external forcing mechanism Pr is equal to 0, 0.3, 0.4, respectively. In this figure, blue represents cooperators, in contrast, red stands for defectors. Cooperators always tend to form a block structure, however, they cannot defend the invasion of defectors when the external forcing mechanism is missing. Similar to that, cooperators and defectors finally form a confrontational situation under a certain strength of external forcing (i.e., situation shown in the center of the figure). As the strength of external forcing raises, the defectors get weaker and weaker and vanish at the end.

temptation to defect $T = b$ under different external forcing strengths when $K = 0.1$. The proportion of cooperation ρ_c decreases rapidly as $T = b$ increases, where the external forcing mechanism is absented. When we introduce the external forcing mechanism, we can find that with the increase of intensity of external forcing mechanism affects the proportion of cooperation in the same $T = b$ case has been increased. In short, the introduction of external forcing mechanism could promote the level of cooperation.

Then, we study the influence of the environment noise factor K on the cooperation level. When the environment noise is huge, namely, when $K \to \infty$, agents could not judge the circumstances precisely for choosing the best strategy rationally, and they would change their strategy stochastically. On the contrary, when $K \to 0$, which means the environment is clear, and agents will update their strategy accurately. The results of the relationship between the level of cooperation ρ_c and K are researched under different intensities of external force Pr. Results represented in Fig. 2 illustrate the proportion of cooperators ρ_c with temptation T for different external force Pr. The results show the change of the cooperation ratio in the steady state with K when $b = 1.5$. The external force strength of 0.1, 0.2, and 0.3 is used in the panels. The cooperation proportion increases with the increase in noise level

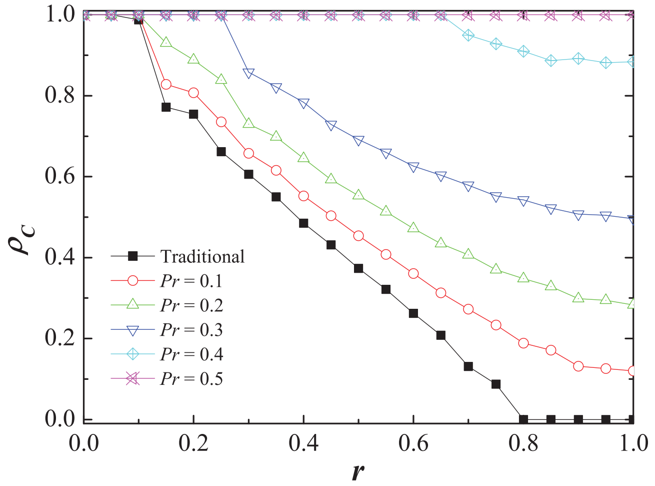

FIG. 5. Proportion of cooperators ρ_c on the square lattice changes with r in the snowdrift game, as acquired with different intensities and without external forcing mechanism. Presented results are obtained for $K = 0.1$.

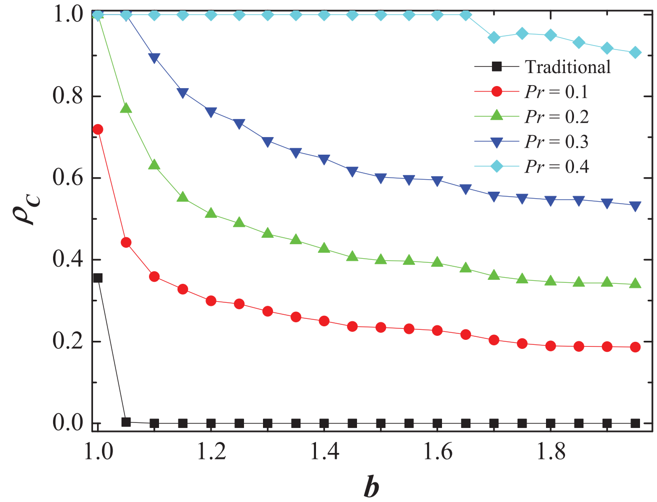

FIG. 6. Proportion of cooperators ρ_c on the square lattice changes with temptation to defect b in the multigames, as acquired with different intensities and without external forcing mechanism. Presented results are obtained for $K = 0.1$, $\Theta = 0.4$. As the external forcing strength get stronger, the temptation lose its influence gradually.

when players are in a high temptation to defect (i.e., $b = 1.5$ as shown in the panel). The reason for this phenomenon is that insufficient external forcing intensity is not enough to inhibit rational groups from choosing defection strategies, and raising the noise level can weaken rational thinking and make it easier to increase the proportion of cooperation. When temptation $T = b$ and external forcing intensity ρ_c are relatively small, the agent's choice of cooperation or defection has little difference in revenue, the external forcing effect is not enough to control the overall situation, Addition of the noise makes the irrational situation appear, and the agent's strategy choices are in great randomness. The worst cooperation ratio occurs when K is about 0.1. The proportion of cooperation is inhibited somehow. This conclusion can guide us to analyze the actual environment in reality and adopt different measures to maintain the cooperation ratio to an ideal level under special circumstances.

Since the introduction of external forcing mechanism promotes the level of cooperation, it is meaningful to detect the potential reason for this phenomenon. In order to analyze the inherent nature of this promotion, we describe the density of cooperation under the time series by plotting the changes of proportion in Fig. 3. In the first few steps, the defectors are in a dominant position to the cooperators. In fact, we know that choosing defection is actually a strategy of potential inclination for every agent because defectors have a high payoff value in one game. As time goes, the power of cooperators will gradually be eroded by defectors and reach the lowest point at some point. Cooperative strategy will gradually disappear, for it could not withstand the temptation of high interest as a defector without external forcing mechanism. The level of cooperation would be maintained at a certain value with weak external forcing. The temptation of defection cannot compete with the global control force with intensive external forcing, and the proportion of cooperation will gradually rise to a certain level from the beginning.

From the pattern figure (Fig. 4), blue represents cooperators and red represents defectors, we can see that cooperators form clusters to spread and resist the invasion of defectors in the process of evolution. On the contrary, defectors do not have the ability to form clusters. The involvement of external forces can maintain the agent in the core position to adopt a cooperative strategy better, as a result, the advantages of partners would be further strengthened. By analogy, with the introduction of external forcing mechanism in scale-free network, and they occupy the hub nodes of interaction network, the cooperators get together fast to decrease the invasion of the defectors. Thereby, the network will form a leader-follower model relationship which will promote the cooperation efficiently.

At last, in order to verify the robustness of the proposed mechanism, simulations are performed with the snowdrift game and multigames. The proportion of cooperators ρ_c in dependence on r with regard to the snowdrift game is shown in Fig. 5. We can find the same law of change of proportion of cooperators as r increased. The external forcing mechanism performs well in the snowdrift game for the proportion of cooperation has been all one at the control ratio of 0.5, which would not happen in weak prisoner's dilemma. Cooperation is easier to exist in the snowdrift game, which is consistent with previous findings. Figure 6 features the proportion of cooperators as a function of temptation to defect $T = b$ in evolutionary multigames. Similar to above results, there exists a range of temptation to defect b insuring the larger proportion of cooperators with setup of external forcing mechanism under different values of Θ. But, we can also find that with the same intensity of external forces, the proportion of cooperation of multigames is higher than that of prisoner's dilemma for an obvious reason that multigames introduce heterogeneity.

IV. CONCLUSION

To summarize, we propose an external forcing mechanism to urge agents learning the cooperative strategy, which can boost the cooperation into a higher level. Specifically, we compare the level of cooperation under different intensities of external forcing with the traditional model, in which agents have the probability Pr $(0 \leq Pr \leq 1)$ to learn cooperative strategy and then have the probability $1 - Pr$ to learn the strategy of his neighbor. The numerous simulation results show that cooperative level can be largely promoted when the mechanism is introduced in prisoner's dilemma. Meanwhile, the time course of the proportion of cooperators and the relationship between cooperative level and uncertainty of the strategy selection are exhibited as well. The reason for the promotion of cooperative strategy is the introduction of external forcing mechanism, which motivates the occurrence of compact cooperative clusters. Cooperators could get together and form clusters quickly so as to resist the invasion of defectors. In addition, we simulate the mechanism under the snowdrift game and multigames for researching the effect of external forcing mechanism deeply. It is clear that the external forcing mechanism indeed promotes the cooperation level.

According to the results, external forcing mechanism is an important method to promote the cooperation level. This mechanism could be extended to other evolutionary games and may achieve high cooperative level efficiently. Conclusions from our experiments are of great practical significance, and it may help us to find the optimal mechanism for maintaining the level of group cooperation under specific circumstances. In addition, there are also broad research directions such as analysis and control of cooperative strategy in terms of different network topologies.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 61703439 and 61374156), in part by the Innovation-Driven Plan in Central South University, and in part by the 111 Project (B17048).

[1] X. Qian, F. Xu, J. Yang, and J. Kurths, "The expansion of neighborhood and pattern formation on spatial prisoner's dilemma," Chaos Interdiscip. J. Nonlinear Sci. **25**, 043115 (2015).

[2] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, "Statistical physics of human cooperation," Phys. Rep. **687**, 1–51 (2017).

[3] C. Zhou and J. Kurths, "Hierarchical synchronization in complex networks with heterogeneous degrees," Chaos Interdiscip. J. Nonlinear Sci. **16**, 015104 (2006).

[4] Y. Sun, J. Kurths, and M. Zhan, "Power-functional network," Chaos Interdiscip. J. Nonlinear Sci. **27**, 083116 (2017).

[5] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," Phys. Rep. **424**, 175–308 (2006).

[6] M. E. Newman, "The structure and function of complex networks," SIAM Rev. **45**, 167–256 (2003).

[7] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," Science **286**, 509–512 (1999).

[8] R. Cohen and S. Havlin, *Complex Networks: Structure, Robustness and Function* (Cambridge University Press, 2010).

[9] M. Newman, *Networks: An Introduction* (Oxford University Press, 2010).

[10] G. Szabó and G. Fath, "Evolutionary games on graphs," Phys. Rep. **446**, 97–216 (2007).

[11] W. Maciejewski, F. Fu, and C. Hauert, "Evolutionary game dynamics in populations with heterogenous structures," PLoS Comput. Biol. **10**, e1003567 (2014).

[12] M. A. Nowak, A. Sasaki, C. Taylor, and D. Fudenberg, "Emergence of cooperation and evolutionary stability in finite populations," Nature **428**, 646–650 (2004).

[13] M. A. Nowak, "Five rules for the evolution of cooperation," Science **314**, 1560–1563 (2006).

[14] E. Pennisi, "How did cooperative behavior evolve?," Science **309**, 93–93 (2005).

[15] G. Liu, H. Shen, and L. Ward, "An efficient and trustworthy p2p and social network integrated file sharing system," IEEE Trans. Comput. **64**, 54–70 (2015).

[16] R. Axelrod and W. D. Hamilton, "The evolution of cooperation," Science **211**, 1390–1396 (1981).

[17] E. Fehr and S. Gachter, "Cooperation and punishment in public goods experiments," Am. Econ. Rev. **90**, 980–994 (2000).

[18] M. Perc, "Chaos promotes cooperation in the spatial prisoner's dilemma game," Europhys. Lett. **75**, 841 (2006).

[19] M. A. Nowak and R. M. May, "Evolutionary games and spatial chaos," Nature **359**, 826–829 (1992).

[20] M. Perc, "Double resonance in cooperation induced by noise and network variation for an evolutionary prisoner's dilemma," New J. Phys. **8**, 183 (2006).

[21] F. C. Santos and J. M. Pacheco, "Scale-free networks provide a unifying framework for the emergence of cooperation," Phys. Rev. Lett. **95**, 098104 (2005).

[22] J. Vukov and G. Szabó, "Evolutionary prisoner's dilemma game on hierarchical lattices," Phys. Rev. E **71**, 036133 (2005).

[23] F. Fu, X. Chen, L. Liu, and L. Wang, "Social dilemmas in an online social network: The structure and evolution of cooperation," Phys. Lett. A **371**, 58–64 (2007).

[24] S. Boccaletti, G. Bianconi, R. Criado, C. I. Del Genio, J. Gómez-Gardenes, M. Romance, I. Sendina-Nadal, Z. Wang, and M. Zanin, "The structure and dynamics of multilayer networks," Phys. Rep. **544**, 1–122 (2014).

[25] K. Huang, Y. Cheng, X. Zheng, and Y. Yang, "Cooperative behavior evolution of small groups on interconnected networks," Chaos Solitons Fractals **80**, 90–95 (2015).

[26] Z. Wang, L. Wang, A. Szolnoki, and M. Perc, "Evolutionary games on multilayer networks: A colloquium," Eur. Phys. J. B **88**, 124 (2015).

[27] Q. Jin, L. Wang, C.-Y. Xia, and Z. Wang, "Spontaneous symmetry breaking in interdependent networked game," Sci. Rep. **4**, 4095 (2014).

[28] C. Hilbe, A. Traulsen, and K. Sigmund, "Partners or rivals? Strategies for the iterated prisoner's dilemma," Games Econ. Behav. **92**, 41–52 (2015).

[29] K. Huang, X. Zheng, and Y. Su, "Effect of heterogeneous sub-populations on the evolution of cooperation," Appl. Math. Comput. **270**, 681–687 (2015).

[30] G. Szabó and C. Töke, "Evolutionary prisoner's dilemma game on a square lattice," Phys. Rev. E **58**, 69 (1998).

[31] K. Huang, X. Chen, Z. Yu, C. Yang, and W. Gui, "Heterogeneous cooperative belief for social dilemma in multi-agent system," Appl. Math. Comput. **320**, 572–579 (2018).

[32] M. Perc and M. Marhl, "Evolutionary and dynamical coherence resonances in the pair approximated prisoner's dilemma game," New J. Phys. **8**, 142 (2006).

[33] Z. Wang, L. Wang, Z.-Y. Yin, and C.-Y. Xia, "Inferring reputation promotes the evolution of cooperation in spatial social dilemma games," PLoS One **7**, e40218 (2012).

[34] Z. Wang, M. Jusup, L. Shi, J.-H. Lee, Y. Iwasa, and S. Boccaletti, "Exploiting a cognitive bias promotes cooperation in social dilemma experiments," Nat. Commun. **9**, 2954 (2018).

[35] J. Qin, Y. Chen, W. Fu, Y. Kang, and M. Perc, "Neighborhood diversity promotes cooperation in social dilemmas," IEEE Access **6**, 5003–5009 (2017).

[36] Z. Wang, A. Szolnoki, and M. Perc, "Different perceptions of social dilemmas: Evolutionary multigames in structured populations," Phys. Rev. E **90**, 032813 (2014).

[37] F. Fu, C. Hauert, M. A. Nowak, and L. Wang, "Reputation-based partner choice promotes cooperation in social networks," Phys. Rev. E **78**, 026117 (2008).

[38] J. A. Cuesta, C. Gracia-Lázaro, A. Ferrer, Y. Moreno, and A. Sánchez, "Reputation drives cooperative behaviour and network formation in human groups," Sci. Rep. **5**, 7843 (2015).

[39] M.-h. Chen, L. Wang, S.-w. Sun, J. Wang, and C.-y. Xia, "Evolution of cooperation in the spatial public goods game with adaptive reputation assortment," Phys. Lett. A **380**, 40–47 (2016).

[40] X. Li, M. Jusup, Z. Wang, H. Li, L. Shi, B. Podobnik, H. E. Stanley, S. Havlin, and S. Boccaletti, "Punishment diminishes the benefits of network reciprocity in social dilemma experiments," Proc. Natl. Acad. Sci. **115**, 30–35 (2018).

[41] M. Perc and A. Szolnoki, "Coevolutionary games–a mini review," BioSystems **99**, 109–125 (2010).

[42]M. H. Vainstein, A. T. Silva, and J. J. Arenzon, "Does mobility decrease cooperation?," J. Theor. Biol. **244**, 722–728 (2007).
[43]T. Hadzibeganovic and C.-y. Xia, "Cooperation and strategy coexistence in a tag-based multi-agent system with contingent mobility," Knowl. Based Syst. **112**, 1–13 (2016).
[44]Z. Wang, L. Wang, and M. Perc, "Degree mixing in multilayer networks impedes the evolution of cooperation," Phys. Rev. E **89**, 052813 (2014).
[45]M. Perc and A. Szolnoki, "Social diversity and promotion of cooperation in the spatial prisoner's dilemma game," Phys. Rev. E **77**, 011904 (2008).
[46]M. Perc, J. Gomezgardenes, A. Szolnoki, L. M. Floria, and Y. Moreno, "Evolutionary dynamics of group interactions on structured populations: A review," J. R. Soc. Interface **10**, 20120997–20120997 (2013).